FURTHER DEVELOPMENT OF THE MODEL OF A FILTRATION FLOW WITHIN THE CONCEPT OF THE EFFECTIVE DIAMETER

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Extension of the concept of the effective diameter to steady-state filtration flows allows us to suggest a generalized ideal model of a porous medium combining all the existing models (models of nonintersecting cylindrical capillaries, of parallel plane pores, etc.) and possible calculation procedures based on the notions of a filtering medium in the form of longitudinal parallel channels with a certain shape of the cross section. Results of a study of a filtration flow are presented for a porous medium. The study was carried out using the calculation model suggested.

Determination of the profit is achieved upon extension of the concept of the effective diameter [1] to isothermal steady-state filtration flows of a viscous Newtonian liquid. Whereas, in the case of using the hydraulic diameter d_{h} as a characteristic dimension, the equations of the Darcy coefficient $\lambda_{d\text{h}} = A/\text{Re}_{d\text{h}}$ are a family of congruent straight lines on the plane $\lambda_{d\text{h}}-\text{Re}_{d\text{h}}$ in the logarithmic anamorphosis (Fig. 1a), in the case of using the effective diameter as a geometric scale

$$d_{\text{e}} = (A_d/A)^{0.5}\, d_{\text{h}} = K_1 d_{\text{h}} \tag{1}$$

the relations $\lambda_{d\text{e}} = f(\text{Re}_{d\text{e}})$ (where $\lambda_{d\text{e}} = K_1 \lambda d_{\text{h}}$ and $\text{Re}_{d\text{e}} = K_1 \text{Re}_{d\text{h}}$ are the modified Darcy coefficient and the modified Reynolds number) are described by the single straight line $\lambda_{d\text{e}} = A_d/\text{Re}_{d\text{e}}$ on the plane $\log \lambda_{d\text{e}}-\log \text{Re}_{d\text{e}}$ (Fig. 1b). This means that in a laminar flow use of d_{e} in the definition of the modified Reynolds number and simplex $\bar{l} = l/d_{\text{e}}$ (the relative length) ensures strict similarity of the flows in open channels or noncircular pressure ducts to the flow in a circular pressure tube in the sense that it allows absolutely accurate hydraulic calculations of pressure and gravity flows from the formulas for a circular pressure tube with d_{e} substituted for d in them:

$$\Delta E = A_d l v^2/(2d_{\text{e}}\, \text{Re}_{d\text{e}}), \tag{2}$$

$$\text{Re}_{d\text{e}} = v d_{\text{e}}/\nu \le \text{Re}_d^{\text{l.cr}} = 2320. \tag{3}$$

When using formulas (2) and (3) for experimental investigation of laminar filtration, the parameter d_{e} can be considered as an effective diameter of pores or a translating element in the porous medium, and the calculation model itself can be reasonably called a generalized ideal model of the porous medium since the shape of the cross-section of parallel prismatic capillaries is not specified and can be arbitrary (for example, with the cross-section as a cylinder, a flat slot, an equilateral triangle, a square, an ellipse, a trigonal or tetragonal asterisc, i.e., translating elements that are formed in the dense packing of circular rods located at the angles of the equilateral triangle or a square, etc.).

For a laminar filtration flow from the Darcy formula (2) describing the hydraulic resistance of laminar pressure and gravity flows with $i = \Delta E/(lg)$ and $v_{\text{f}} = vm$, the following relation is obtained to be used for calculating the effective pore diameter:

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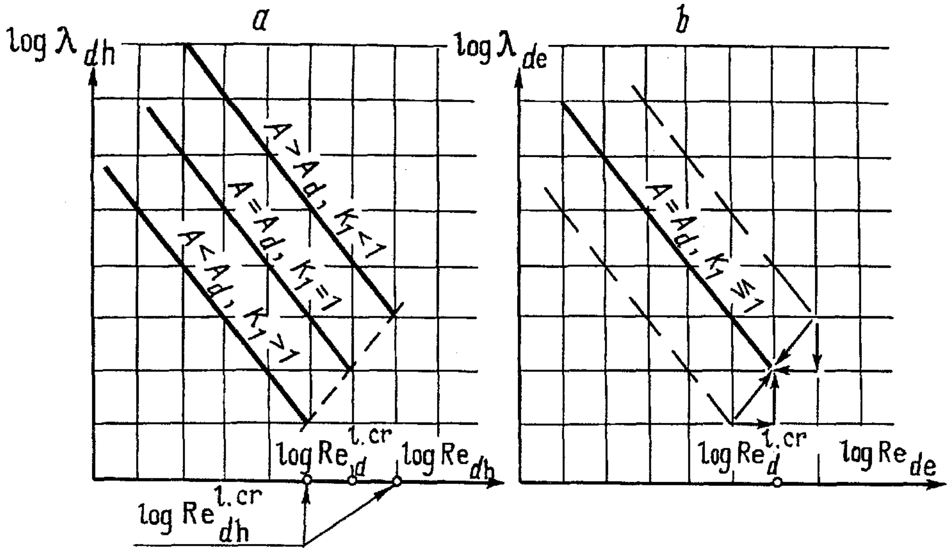

Fig. 1. Shapes of the equations of the Darcy coefficient λ_L in noncircular ducts and channels in the zone of laminar flow when hydraulic (a) and effective (b) diameters are used as a characteristic dimension L.

$$d_e = [A_d \nu v_f/(i2mg)]^{0.5} . \tag{4}$$

From relation (4) with $K = v_f\nu(ig)$ the following formula can be obtained:

$$d_e = (0.5\, A_d K/m)^{0.5} , \tag{5}$$

which relates the effective pore diameter to the permeability coefficient of the porous medium $K = C\nu/g$, where C is the filtration coefficient entering into the linear relation suggested by Darcy

$$v_f = Ci . \tag{6}$$

If this relation, which in the form of the Poiseuille relation for pressure and gravity flows has the form $v_f = mv = m\Delta E 2d_e^2/(l\nu)$, were observed strictly, then without clogging of the pores, every porous medium could be characterized in terms of constant values of d_e or K. As can be seen from numerous experimental data [2], the hypothetic formula (6) should be recognized as invalid. In laminar filtration, because of the complexity of the filtration process itself (unsteady flow, differences in individual characteristics of the porous media, presence of numerous concurrent hydrodynamic phenomena [2], etc.), the coefficients or parameters that characterize integral properties of the porous medium cannot and must not remain constant in the range $\mathrm{Re}_{de}=0-2320$. All the aforementioned statements are confirmed indirectly by the following parameters. In the case of stabilized curvilinear liquid flow that is more complicated in comparison with stabilized flows in tubes and channels and less simple in comparison with filtration at a fixed curvature parameter (the ratio of the diameter of the coil to the diameter of the tube), with increase in the Reynolds number Re_d, the following resistance zones are changed [3]: the zone of laminar flow, where curvilinearity of the flow has no effect on the hydraulic resistance; the zone of laminar flow with increasing macrovortices, where the resistance is affected not only by the Reynolds number but also by the curvature parameter; the zone of transient flow with macrovortices; the zone of a turbulent flow with increasing damping of macrovortices, where the hydraulic resistance is affected by the Reynolds number and curvature; the zone of turbulent flow, where the curvature no longer affects the resistance. In nonstabilized flow in valve devices that is simpler than a filtration flow, five zones of hydraulic resistance are distinguished in [4], depending on the character of the effect of the Reynolds number Re_d on the coefficient ζ: in the first zone $\zeta = B/\mathrm{Re}_d$, where the flow is laminar in the pipeline and in the local resistance; in the second zone $\zeta = D/\mathrm{Re}_d^x$, where the laminar flow is violated in the local resistance; in the third zone $\zeta = \mathcal{E}\mathrm{Re}_x^{0.53}$, where the laminar flow is violated in the pipeline too; in the fourth and fifth zones $\zeta \cong F$, where the Reynolds number is low or has no effect on the coefficient ζ at all.

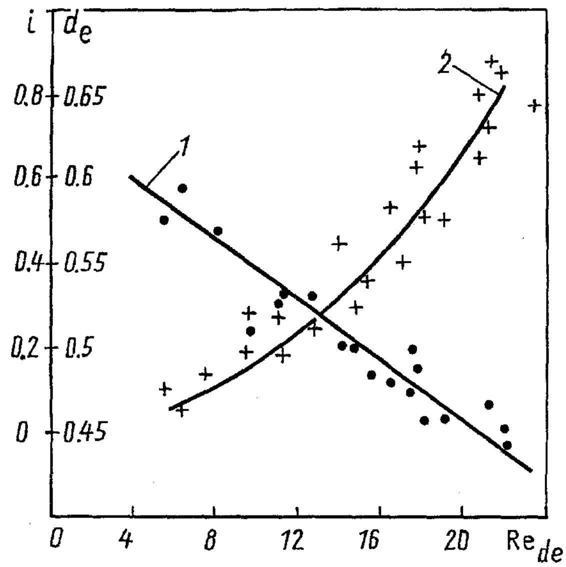

Fig. 2. Plot of the effective pore diameter d_e, mm (1) and the dimensionless slope i (2) versus the Reynolds number Re_{de} in the case of filtration of water through a filling of polystyrene particles; $d_e = a_0 + a_1 Re_{de}$, $a_0 = 0.6328$, $a_1 = -8.994 \cdot 10^{-3}$; $i = a_2 Re_{de}^2$, $a_2 = 1.633 \cdot 10^{-3}$.

In the present work the parameter d_e, complex according to (1) and calculated from formula (4), is used to characterize the filtering porous media. Its use as a geometric scale does not require specification of the shape of parallel prismatic capillaries. This is very important as the relation between A and the shape of the cross section is not single-valued [1].

Existing ideal models are particular cases of the generalized model suggested. At $K_1 = 1$, $K_1 = (2/3)^{0.5}$, etc., the generalized model becomes the known models of nonintersecting cylindrical capillaries ($A = A_d$, $d_e = d$), parallel plane slot pores ($A = 96$), etc. The generalized and partial models of the porous medium differ in approaches to correlating the characteristics d_e and d_h to the structure of the medium. For example, in the case $K_1 = 1$, d evaluated from formula (4) at $d_e = d$ is correlated to the pore diameter d_p determined by the method of displacement of liquid from the pores or pressing mercury into the pores. It should be noted that because d_p does not coincide with d ($d_p > d$) in generalization of experimental data the parameter d_p is preferred but, unfortunately, it does not allow the results of investigations [2] to be considered as being obtained within the generalized ideal model by simple substitution of d_e for d according to Eq. (4).

According to definition (1), the complex parameter d_e includes both the geometry of the cross-section (in terms of d_h) and the hydraulic resistance (A in terms of K_1). Thus, in the generalized model it is possible to evaluate A from formula (1) in terms of d_e if it is technically feasible to determine the hydraulic diameter d_h averaged on a certain length. The possibility of its calculation for porous media is determined by the possibility of determining the length-average total pore diameter within a real or mentally isolated (not too small) free cross-sectional area since determination of the area of the pores themselves is not difficult: the cross-sectional area S is multiplied by the clearness coefficient n which is usually assumed to be equal to the porosity m. If processing of experimental data is completed by determination of A, this means that, eventually, d_e is correlated to both the structure of the porous medium in terms of d_h and to its main property, the hydraulic resistance, in terms of the coefficient K_1: $A = A_d K_1^{-2}$.

The physical meaning of the effective pore diameter d_e is clearer (it includes both the geometry of the porous medium and its hydraulic resistance) than the physical meaning of the permeability coefficient K or filtration coefficient C. Note also that the deviation of the diameter d_e from its average value within the range of the numbers Re_{de} is always smaller than the deviation of the coefficients K and C.

Because of the complexity of the filtration process and the diversity of structures of porous media (porous rocks, fillings with particles equal and unequal in size and shape, porous media in the form of products from metal powders, fibers, and networks), obtaining a universal relation in the form $A = f(\mathrm{Re}_{dh})$ seems unlikely. Under these conditions it is most reasonable and pragmatic to study the behavior of the diameter d_e as a function of the number Re_{de} in the feasible range $0...\mathrm{Re}_{de} \le 2320$ since approximation of experimental data by the relation $d_e = f(\mathrm{Re}_{de})$ will be more objective and less labor-consuming in comparison with their representation in the form of the indirect relation $A = f(\mathrm{Re}_{dh})$, although the latter is more interesting.

As an example, in Fig. 2 data are presented on the hydraulic resistance of a filtration flow processed within the generalized ideal model of porous media suggested. A pipeline with the diameter 22.4 mm was filled with granulated polystyrene in the form of elliptic (with semiaxes of ~1 mm and ~1.5 mm) cylinders ~3 mm in height, providing $m = 0.4$; a constant level of water in the feeding tank and its closed circulation in the experimental setup provided steadiness and isothermicity of pressure filtration; the presence of filling before the first piezometer and after the second piezometer prevented the effect of input and output losses of the specific hydraulic energy. One should note that at $\mathrm{Re}_{de} = 2320$ with the average $d_e = 4.91 \cdot 10^{-4}$ m for the experimental conditions ($l = 1.0$ m; $\nu = 1.136 \cdot 10^{-6}$ m^2/sec, and $n \cong m = 0.4$), the losses of the specific hydraulic energy (or the available specific potential energy) will be 808 J/kg, which indicates that at Re_{de} approaching even the lower critical value $\mathrm{Re}_d^{1,\mathrm{cr}}$ organization of experiments is practically impossible.

It seems likely that using the effective diameter (which is simultaneously a characteristic of porous medium) as a characteristic dimension has the same value as the use of the hydraulic diameter d_h by hydraulic engineers as a geometric scale for noncircular ducts and channels. In this case it is unnecessary to recalculate the results of processing in the form of criterial relations for filtrations due to using different geometric scales or their different values both within models (ideal and physical ones [2] based on the presentations of porous medium in the form of prismatic capillaries and various types of local resistances) and in transition from one model to another.

NOTATION

l, length; g, acceleration due to gravity; v and v_f, true and fictitious average velocities; ΔE, friction losses of specific hydraulic energy along the length (or available specific potential energy); λ_L and ζ, Darcy and Weisbach coefficients; $\mathrm{Re}_L = vL/\nu$, Reynolds number; ν, kinematic viscosity; L, characteristic linear dimension; d, diameter; $d_h = 4S/P$, hydraulic diameter; S, cross-sectional area; P, wetted perimeter; $d_e = K_1 d_h$, effective diameter; $K_1 = (A_d/A)^{0.5}$, shape factor; $A = \lambda_L \mathrm{Re}_L$, hydraulic resistance at $L = d_h$ ($A = A_d = 64$ at $L = d$ and $L = d_e$); K and C, permeability coefficient and filtration coefficient; $\mathrm{Re}_d^{1,\mathrm{cr}} = 2320$, lower critical Reynolds number at $L = d$ and $L = d_e$; $i = \Delta E/(lg)$, dimensionless slope; m and n, porosity (the ratio of the pore to medium volumes) and clearness coefficient (the ratio of the pore to medium areas).

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